Middle Years Programme

Markscheme

May 2023

Mathematics

On-screen examination

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

The markscheme may make use of the following abbreviation: OE – 'or equivalent'

RM Assessor

Important   **NR** only use when the candidate has not made any response also stamp the response with **SEEN** Do not award **A0** for NR responses

RM Assessor has the following annotations that should be used to award marks:

Annotation	Explication	Annotation	Explication
?	Unclear	A0	Award 0 marks
SC	Special case	✓1	Award 1 mark
MR	Misread	✓2	Award 2 marks
NWS	No working shown	✓3	Award 3 marks
ECF	Error carried forward	✓4	Award 4 marks
WTTE	Words to that effect	✓5	Award 5 marks
BOD	Benefit of the doubt	✗	Incorrect
AG	Answer Given	(highlight)	Highlight tool
(wavy underline)	Wavy underline tool	SEEN	Seen
○	Ellipse tool	(comment)	On page comment tool

The markscheme abbreviations:

• Bullet notation means award 1 mark – see example below

Example 1
.1 mark awarded and corresponding notes are aligned

b	.1 Show clear line of reasoning in the method .2 4	.1 45 and 49 seen **OE** *Ex:* $49 = 45 + x$ .2	**2**

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.

b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.

c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.

d) To award **ECF** marks for a question part, **there must be working present for that part**.

e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.

f) A misread (**MR**) is an error**. ECF** is normally awarded.

General points
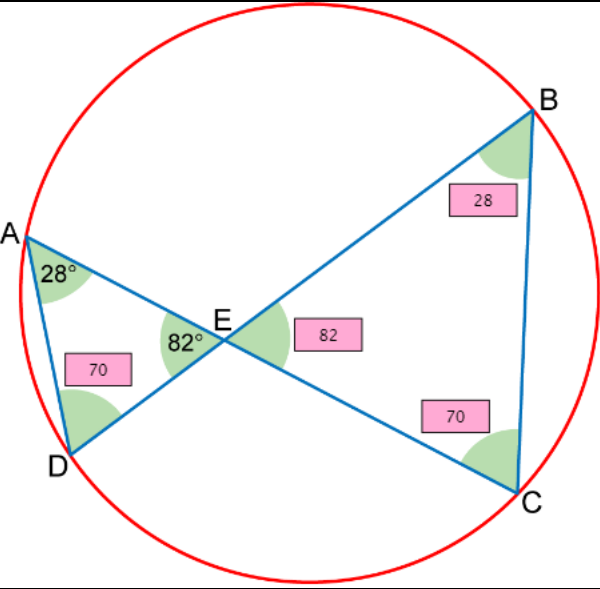
a) As this is an international examination, accept all **alternative forms of notation,** for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS.

b) **ACCEPT** notation errors in intermediate steps.

c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer,** then that last mark cannot be awarded.

d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.

e) Where candidates have written two solutions to a question, mark the first solution.
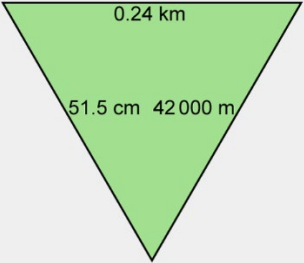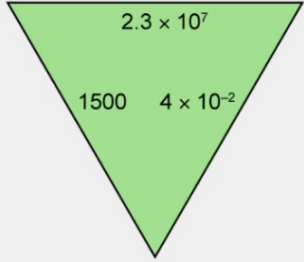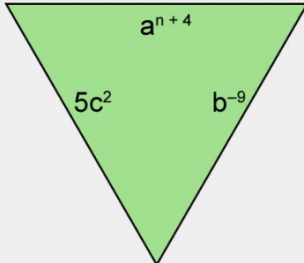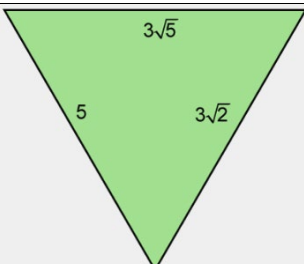
f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ **OR** 1/2 **OR** 0.5 **OR** $2 \div 4$; $\frac{x}{2}$ **OR** $x / 2$ or $x \div 2$; 0.23 **OR** 23%

g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, it indicates what the candidate's result represents. Ex: if last mark is for the result: (AB)=5; this means we award the mark for seeing 5 as the result of calculating AB without necessarily seeing AB=5, but it does not mean we award the mark for seeing 5 representing another length

h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.

i) Accept seeing equation not in-line.

j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.

k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.

l) **ACCEPT** using the correct values or working regardless their previous result.

m) Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. **If in doubt, contact your team leader for advice.**

n) Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says "4.3(3…) without working award 2 marks", then seeing any acceptable working step and seeing 4.3(3…) as the answer award the 3 marks.

o) For "**show that**" questions, unless otherwise noted, every bullet point has to be seen in order to be awarded.

Q1		Answers	Notes	Total
	a	Place the correct triangle	0.24 km / 51.5 cm 42 000 m	1
	b	Place the correct triangle	2.3×10^7 / 1500 4×10^{-2}	1
	c	Place the correct triangle	a^{n+4} / $5c^2$ b^{-9}	1
	d	Place the correct triangle	$3\sqrt{5}$ / 5 $3\sqrt{2}$	1

Q2		Answers	Notes	Total
a		.1 Correct numerators for all branches .2 Correct denominators for all branches	ACCEPT $\frac{3}{8}$ $\frac{9}{24}$ D $\frac{10}{25}$ D $\frac{15}{24}$ W $\frac{10}{24}$ D $\frac{15}{25}$ W $\frac{14}{24}$ W ACCEPT $\frac{7}{12}$	2
b		.1 Multiply their $P(D)$ by their $P(D\mid D)$ from a) .2 Correctly calculate their result after multiplying their $P(D)$ and their $P(D\mid D)$ from a)	.1 $\dfrac{their10}{25} \times \dfrac{their9}{their24}$ .2 their $\dfrac{3}{20}(= 0.15)$ OE	2

mathmmoeengtz0xxm

c	**AM1** .1 Subtract their b) from 1 .2 Correct result after subtracting their b) from 1	**AM1** .1 $1 - \text{their}\dfrac{3}{20}$ OE ACCEPT 100-their15 .2 $\text{their}\dfrac{17}{20}(= 0.85)$ OE	
	AM2 .1 Correctly write their three products of probabilities for their three cases from a) DW,WD,WW .2 Correctly calculate their result after adding their probabilities for at least <u>two</u> cases from DW,WD,WW	**AM2** .1 $\dfrac{\text{their}10}{25} \times \dfrac{15}{\text{their}24}$ OE and $\dfrac{\text{their}15}{25} \times \dfrac{10}{\text{their}24}$ OE and $\dfrac{\text{their}15}{25} \times \text{their}\dfrac{14}{24}$ OE or their0.25 and their0.25 and their0.35 .2 $\text{their}\dfrac{17}{20}(= 0.85)$ OE .2 ACCEPT only if their result includes at least two cases from their DW,WD,WW from a)	**2**

d			
	.1 Correct three probabilities from their a) for the case DDW in any order, <u>with OR without replacement</u>	.1 $\dfrac{\text{their}10}{25}$, $\dfrac{\text{their}10}{25}$, $\dfrac{15}{25}$ OR $\dfrac{\text{their}10}{25}$, $\dfrac{\text{their}9}{24}$, $\dfrac{15}{23}$ .1 ACCEPT $\dfrac{12}{125}(=0.096)$OE OR $\dfrac{9}{92}(=0.0978(26...)\text{or }0.098$ OE	
	.2 Multiply <u>their</u> $P(D\cap D\cap W)$, in any order , <u>with OR without replacement</u>, by 3 (or add the same 3 times)	.2 $\quad 3\times\dfrac{\text{their}10}{25}\times\dfrac{\text{their}10}{25}\times\dfrac{15}{25}$ OE OR $\quad 3\times\dfrac{\text{their}10}{25}\times\dfrac{\text{their}9}{24}\times\dfrac{15}{23}$ OE OR $\quad 3\times$ their bp1	**3**
	.3 Correctly calculate their result, including their $P(D\cap D\cap W)$ in any order, added at least <u>twice</u> <u>without replacement</u>	.3 their $\dfrac{27}{92}(=0.29(3478...))$ OE ACCEPT 0.3 .3 ACCEPT only if .1 or .2 are awarded without replacement .3 DO NOT ACCEPT their 0.29(3478…) that comes from incorrect rounding in bp1 or bp2	

Q3		Answers	Notes	Total
	a	.1 Correctly write two values .2 Correctly write another two values		**2**
	b	.1 Correctly state the reason	<u>Any two from:</u> DAE = CBE ACCEPT A=B ADE = BCE ACCEPT D=C AED = BEC ACCEPT E the same or common E WTTE ACCEPT they both have equal or same or congruent angles WTTE ACCEPT they share equal angles or they share same angles Ignore additional incorrect reasons DO NOT ACCEPT they share 3 angles DO NOT ACCEPT they have similar angles or corresponding angles WTTE DO NOT ACCEPT sides have ratio or factor of 4 WTTE DO NOT ACCEPT they are the case AAA or angle-angle –angle	**1**

c			c
	.1 Correctly apply the similarity	.1 $\dfrac{1}{x+1}=\dfrac{x-1}{x+5}$ OE ACCEPT $\dfrac{DE}{EC}=\dfrac{AE}{EB}$ OE	
	.2 Correctly cross multiply their ratios in terms of x	.2 their $(x+5)=$ their $(x+1)(x-1)$ OE ACCEPT not seeing this step, .2 Seeing $(x+5)=(x+1)(x-1)$ or $(x+5)=x^2-1$ implies .1	
	.3 Correctly rearrange their quadratic in .2 to one side .4 Correctly factorize $x^2-x-6(=0)$ OR correctly substitute the correct coefficients into the quadratic formula (x=)3 AG	.3 their $x^2-x-6(=0)$ .4 $(x-3)(x+2)$ ACCEPT $(x=)-2$, $(x=)3$ OR $(x=)\dfrac{1\pm\sqrt{1+4\times6}}{2}$ OE ACCEPT $(x=)\dfrac{1+\sqrt{1+4\times6}}{2}$ OE **For 4 marks .1, .3, .4 or .2, .3, .4 must be seen**	**4**
d	.1 Correct factor seen OR correctly substitute into sine rule OR cosine rule .2 Correctly calculate BC .3 Correct result after adding the 5 correct sides to their8.4	.1 factor is 4 or ratio is 4 or seen in calculation ACCEPT $\dfrac{1}{4}$ OE instead of 4 OR $\dfrac{BC}{\sin82}=\dfrac{8}{\sin70}$ or $\dfrac{4}{\sin28}$ OE OR $(BC^2=)8^2+4^2-2\times8\times4\times\cos82$ OE .2 $(BC=2.1\times4=)$ 8.4 or 8.4(372…) or 8.4(316…) .3 (17.1+their8.4=) their 25.5 OE .3 DO NOT ACCEPT $17.1+x$	**3**

Q4		Answers	Notes	Total
	a	.1 Correct side to use in the right-angle triangle	.1 $\dfrac{24}{2}$ or 12 seen even within working	
		.2 Correctly substitute 12 into a trig ratio	.2 $\tan 55 = \dfrac{a}{12}$ or $(a =)12\tan 55$ or $\dfrac{12}{\tan 35}$ or $\dfrac{a}{\sin 55} = \dfrac{12}{\sin 35}$ OE .2 ACCEPT using cosine or sine ratio then using Pythagoras correctly	
		.3 Correct value of a before rounding 17.1 AG	.3 17.137(77…) ACCEPT 17.13 or 17.14	**3**
	b	**AM1** .1 Correctly substitute into Pythagoras	**AM1** .1 $(h^2) = 17.1^2 - 12^2$ OE or $(h =)\sqrt{17.1^2 - 12^2}$ .1 ACCEPT using 17.13(777…) or 17.14 instead of 17.1	
		.2 Correct value of h	.2 12.18(236...) ACCEPT [12.18,12.24]	
		AM2 .1 Correct angle in the right-angled triangle	**AM2** .1 $(\cos^{-1}(\dfrac{12}{17.1}) =)45.4(32...)$ or $(\sin^{-1}(\dfrac{12}{17.1}) =)44.56(79...)$ or 44.6 .1 ACCEPT 45 only with correct working .1 ACCEPT using 17.137…or 17.14 instead of 17.1	**2**
		.2 Correct value of h	.2 $(12 \times \tan(45.4)$ or $\dfrac{12}{\tan 44.6}$ or $\dfrac{12\sin 45.4}{\sin 44.6}$ OE $=)12.16(87....)$ .2 ACCEPT [12.16,12.24]	

	c	.1 Correctly substitute their b) into volume of pyramid formula	.1 $\frac{1}{3} \times 24 \times 24 \times \text{their}12.18$ or $\frac{1}{3} \times 576 \times \text{their}12.18$ .1 ACCEPT their12.18 from b) being any positive number	**3**
		.2 Correctly calculate their result after substituting into their volume of pyramid formula	.2 their 2338.56 .2 ACCEPT their result only if it includes $24 \times 24 \times \text{their}12.18$ or if it is $\frac{1}{3} \times \text{their area} \times \text{their}12.18$ .2 DO NOT ACCEPT their result if it is $\frac{1}{3} \times \text{length} \times \text{their}12.18$	
		.3 Correctly round their volume to the nearest integer	.3 their 2339 .3 ACCEPT only if bp1 or bp2 is awarded AND they are rounding a non-integer value	
	d	**AM1 (using increased volume)** .1 Correct factor for the length	**AM1 (using increased volume)** .1 "length factor is 1.1" WTTE or seen within one calculation for length Ex: their13.4 or 26.4	**3**
		.2 Correctly calculate their volume using their correctly enlarged lengths	.2 $(\frac{1}{3} \times \text{their}26.4 \times \text{their}26.4 \times \text{their}13.4 =) \text{their}\,3113.(088)$ .2 ACCEPT their volume calculated using early rounding for lengths or area of base	
		.3 Correct percentage increase	.3 33(.1%)	
		AM2 (using only factor) .1 Correct factor for the length	**AM2 (using only factor)** .1 "length factor is 1.1" WTTE or seen within one calculation for length Ex: their13.4 or 26.4	
		.2 Cube their factor for the length	.2 $(\text{their}1.1)^3$ OE or their1.33(1)	
		.3 Correct percentage increase	.3 33(.1%)	

Q5		Answers	Notes	Total
	a	Correct result	$(0.16 \times 4500 =)720(t)$	**1**
	b	.1 Correct ratio	.1 $\frac{3}{8}$ (=0.375) OE or $\frac{8}{3}$ or 3:8 or 8:3 OE or seen within calculations Ex: $\frac{\text{their}720}{2.67}$ OE .1 ACCEPT seeing $\frac{3}{8}$ as 0.4 or $\frac{8}{3}$ as 2.6 or 2.7 in bp1 only .1 DO NOT ACCEPT 3×8 as ratio or seen within calculations	
		.2 Correctly apply <u>the correct</u> ratio on their720	.2 $(\frac{3}{8} \times \text{their } 720$ or $\text{their}720 \div \frac{8}{3} =) \text{their } 270$	**2**

	c	.1 Correctly write two values .2 Correctly write a third values .3 Correctly write a fourth value .4 Correctly write fifth and sixth values	 Answers from (a) and (b) are not counted as one of the six values The six values are: r domestic=0.1 OE ACCEPT 1/10 or "1:10" or "1 to 10" OE r commercial= their r for domestic 2r=(2×their r domestic=)their0.2 ACCEPT their2/10 or their "2:10" or their"2 to 10" OE Energy commercial=$(0.1 \times$ their720=) their72 Energy industrial=$(0.2 \times$ their270=) their54 Total Energy=(135+their72+their54=) their261 ACCEPT Energy Commercial 72 or Energy industrial 54 regardless their r value	**4**
	d	.1 Correct mid-interval values .2 Add the product of their mid-interval values by frequency .3 Divide their sum by 12	.1 860, 880, 900,920, 940 ACCEPT seeing only three correct .2 $2 \times$ their860 $+4 \times$ their880 $+2 \times$ their900 $+1 \times$ their920 $+3 \times$ their940 OE or 10780 .2 ACCEPT seeing only three correct products OE added .3 Ex: $\dfrac{\text{their bp2}}{2+4+2+1+3}$	**4**

For the table inside cell c:

		Weight, tonnes (t)	Production ratio Energy per tonne (TJ / t)	Energy Tera-joules (TJ)
Category	Domestic	1350	r =	0.1 or 1/10 or 1:10 or "1 to 10"	135
	Commercial	Answer from (a) their 720	r =	their 0.1 (above)	(0.1 × their720=) their72
	Industrial	Answer from (b) their 270	2r =	(2 × their 0.1 =) their 0.2 OE	(0.2 × their270=) their54

Total energy produced by the organization this months: (135 + their72 + their54=) their261

		.4 Correct mean before rounding AG 900	.	
	e	.1 Correct ratio seen <u>in any notation</u>	.1 "if 68420 is 900 then their261 is" WTTE or 68420:900 or $\dfrac{68420}{900}$ OE $(=\dfrac{3421}{45}=76.022...)$ ACCEPT 76 .1 ACCEPT 900:68420 or $\dfrac{900}{68420}$ OE $(=\dfrac{45}{3421}=0.013(154...))$	**3**
		.2 Correctly apply <u>the correct</u> ratio on their total energy from c)	.2 $\dfrac{68420}{900}$ OE $\times$ their261 or their261 $\div\dfrac{900}{68420}$ OE or their19841.8 .2 ACCEPT using 76 instead $\dfrac{68420}{900}$ or using 0.013 instead of $\dfrac{900}{68420}$	
		.3 Correctly write their result as an integer	.3 their 19842 .3 ACCEPT their rounding being up or down .3 ACCEPT only if .1 or .2 is awarded	

Q6		Answers	Notes	Total
	a	.1 Correctly substitute coordinates of a point (n,E) from the graph into the equation	.1 Examples: $120 = a(10-0)^2 + 80$, $110 = a(10-1)^2 + 80$ $105 = a(10-2)^2 + 80$, $90 = a(10-5)^2 + 80$,.. etc The points (n,E) from the graph: (0,120), (1 ,112), (2,105), (3,100), (4, 95), (5,90), (6,86), (7,84), (8,82) .1 ACCEPT E being ± 5 the values indicated above .1 ACCEPT $120 - 80$	**2**
		.2 Correct value of a after the substitution of (0,120) or (5,90) a = 0.4 AG	.2 $(a =)\dfrac{40}{100}$ ACCEPT $100a = 40$ or $(a =)\dfrac{10}{25}$ or $\dfrac{2}{5}$ ACCEPT $25a = 10$ .2 DO NOT ACCEPT decimal	
	b	.1 Correctly substitute 2 into the equation .2 Correct answer	.1 $(E =)0.4(10-2)^2 + 80$ ACCEPT $(E =)a(10-2)^2 + 80$ .2 105.6 OE ACCEPT 106 or 105	**2**

	c	.1 Correctly subtract their result in b) from 120	.1 $(120 - \text{their}105.6 =)\text{their}14.4$ .1 ACCEPT their 14.4 rounded up or down Ex: 15 or 14 Ex: $(120 - \text{their}106 =)\text{their}14$ or $(120 - \text{their}105.6 =)\text{their}14$	
		.2 Correct result after multiplying their bp1 by 60 000	.2 $(\text{their}14.4 \times 60000 =)\text{their}\,864\,000$ .2 ACCEPT their b)$\times 60000$=their6 336 000 or 120×60000=7 200 000 .2 ACCEPT their864 000 if their14.4 is rounded up or down Ex: if their bp1 is 14.4 and they use 15 or 14 in bp2	
2				2

d	**AM1**	**AM1**	
	.1 Correctly substitute 90 into the equation	.1 $90 = 0.4(10-n)^2 + 80$	
	.2 Correct rearrangement for $(10-n)^2$ on one side	.2 $(10-n)^2 = \dfrac{90-80}{0.4}(=25)$ OE	
	.3 Correctly square root their both sides of <u>their quadratic</u> in bp2	.3 $\text{their}(10-n) = \text{their} \pm 5$ ACCEPT $\text{their}(10-n) = \text{their}5$	
	.4 Correct identify the positive value of n	.4 $(n=)5$ ACCEPT 2027	
	AM2	**AM2**	
	.1 Correctly substitute 90 into the equation	.1 $90 = 0.4(10-n)^2 + 80$	
	.2 Correct quadratic in the form $ax^2 + bx + c(=0)$	.2 $0.4n^2 - 8n + 30(=0)$ or $n^2 - 20n + 75(=0)$ OE .2 ACCEPT $90 = 0.4n^2 - 8n + 120$	4
	.3 Correctly factorise <u>their quadratic</u> in bp2 or substitute correctly the coefficients of <u>their quadratic</u> into quadratic formula	.3 their $(n-15)(n-5)(=0)$ OR their coefficients in $\dfrac{8 \pm \sqrt{8^2 - 4 \times 0.4 \times 30}}{2 \times 0.4}$ OE .3 ACCEPT one notation mistake in substitution Ex: with their coefficients $\dfrac{8 + \sqrt{8^2 - 4 \times 0.4 \times 30}}{2 \times 0.4}$ or $8 \pm \dfrac{\sqrt{8^2 - 4 \times 0.4 \times 30}}{2 \times 0.4}$ OE	
	.4 Correct identify the positive value of n	.4 $(n=)5$ ACCEPT 2027	
	Next page for AM3 using numbers	Next page for AM3 using numbers	

		AM3 (using numbers)	**AM3 (using numbers ignore additional incorrect substitutions/calculations)**	
		.1 Correctly substitute a number $n \neq 5$ and correctly calculate corresponding E	.1 Ex: $0.4(10 - 1)^2 + 80 = 112.4$ ACCEPT 112	
		.2 Correctly substitute 5 and equate to 90	.2 $0.4(10 - 5)^2 + 80 = 90$	
		.3 Correctly substitute another number $n \neq 5$ and correctly calculate corresponding E	.3 Ex: $0.4(10 - 3)^2 + 80 = 99.6$ ACCEPT 100	
		.4 Correct identify the positive value of n	.4 $(n =)5$ ACCEPT 2027	

Q6e total 10 marks

Mark	1	2		
Factors (F)	**Two** from the keywords/phrases below mentioned in the factors box • filter(s) • carbon or unit(s) or kt or emission(s) • limit(s) or restriction(s) or requirement(s) or regulation(s) or "difference in emission(s)" or "value(s) of difference" DO NOT ACCEPT other similar keywords Ex: "excess" or "reduction" **ACCEPT** multiple keywords in one sentence and ignore additional irrelevant factors	**The three** keywords/phrases below mentioned in the factors box • filter(s) • carbon or unit(s) or kt or emission(s) • limit(s) or restriction(s) or requirement(s) or regulation(s) or "difference in emission(s)" or "value(s) of difference" DO NOT ACCEPT other similar keywords Ex: "excess" or "reduction" **ACCEPT** multiple keywords in one sentence and ignore additional irrelevant factors		

Key
DC: The missing values in the row of Difference in carbon emissions
VC: The missing values in the row of Value of difference in carbon emissions ACCEPT negative values for buying (Ex: -0.48,-0.63,-0.72)

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030
Difference in carbon emissions (kt)	28	19.5	12	5.5	0	4.5	8	10.5	12
Value of difference in carbon emissions ($ million)	1.68	1.17	0.72	0.33		0.27	0.48	0.63	0.72
Buy or sell	sell	sell	sell	sell		buy	buy	buy	buy

Total of Sell is: 1.68+1.17+0.72+0.33=3.9   **Total of Buy is: 0.27+0.48+0.63+0.72=2.1**

Mark	1	2	3	4
Calculate (C)	**One** from: (see table above) • Correctly write **three** DC • Correctly calculate **four** of <u>their</u> VC • Correctly calculate cost if no filters installed: (222x60k=)13.32 ($ mil) **ACCEPT** -13.32 • Weak attempt to calculate <u>their</u> cost if filters installed: They forget the cost of filters or income from selling or price of buying Ex: their$3.9-$their2.1 or their1.8 Ex: $14-$their3.9 or their10.1 Ex: $14+$their2.1 or their16.1	**Two** from: (see table above) • Correctly write **three** DC • Correctly calculate **four** of <u>their</u> VC • Correctly calculate cost if no filters installed : (222x60k=)13.32 ($ mil) **ACCEPT** -13.32 • Good attempt to calculate their cost if filters installed: They include cost of filters, income from selling, and price of buying but incorrect final result Ex: $14+$their$3.9-$their2.1 or their 15.8($mil) **ACCEPT** <u>the</u> correct costs without final result Ex: filters 14 then sell for 3.9 then buy for 2.1 WTTE	**Three** from: (see table above) • Correctly write **all** DC • Correctly calculate **all** of <u>their</u> VC • Correctly calculate cost if no filters installed: (222x60k=)13.32 ($ mil) **ACCEPT** -13.32 • Correctly calculate <u>their</u> cost if filters installed $(14-$their$3.9+$their$2.1=)$their12.2 **ACCEPT** $-$their12.2($mil)	The **four**: (see table above) • Correctly write **all** DC • Correctly calculate **all** of <u>the</u> VC • Correctly calculate cost if no filters installed: (222x60k=)13.32 ($ mil) **ACCEPT** -13.32 • Correctly calculate <u>the</u> cost if filters installed: $(14-3.9+2.1=)12.2$ ($mil) **ACCEPT** -12.2($mil)

Mark	1	2
Recommendation (R)	**ACCEPT only if C1 is achieved** Recommendation with **one weak** justification considering the 8 years period or beyond the 8 years period **Examples considering the 8 years period** Ex: Filters will save money WTTE (and we see values aligned with this recommendation even if incorrectly calculated) Ex: Should install filters but in 4 years they will have to buy new ones WTTE Ex: Should not install filters because after 2026 (or starting 2027) filters will not be efficient enough WTTE Ex: Should not install because it is not economic (and we see values aligned with this recommendation even if incorrectly calculated) Ex: should not install filters because the price of filters is paid upfront WTTE. **DO NOT ACCEPT** seeing only price of filters too high **OR** **Examples considering beyond 8 years period** Ex: After 2030 (or after 8 years), installing filters will be no good or they will have to buy new filters or take new measures WTTE Ex: On the long run (in the future) they will have to buy more filters WTTE **ACCEPT** not recommending filters if their justification supports it **DO NOT ACCEPT** "install filters because it is better for environment" WTTE	**ACCEPT only if C4 is achieved** Recommendation to install filters with **one good** justification aligned with the correct calculations Ex: Should install filters because they will save 1.12 (mil$) WTTE Ex: Should install filters because without filters they will spend 13.32(mil) while with filters only 12.2 WTTE **ACCEPT** Should install filters because it will save money WTTE, and we see 13.32(mil$) and 12.2(mil$) in the calculations

Mark	1	2
Accuracy (A)	Implies inaccurate with **one** justification • Price of buying/selling (60000) carbon units is approximate WTTE OR Limits/restrictions could change (based on world events or to decrease pollution or because climate change). WTTE	Implies inaccurate with **two** justifications • Price of buying/selling (60000) carbon units is approximate WTTE OR Limits/restrictions could change (based on world events or to decrease pollution or because climate change). WTTE **AND** • (Carbon) emissions are predictions/estimates or might change over the years WTTE
	OR • (Carbon) emissions are predictions/estimates or might change over the years WTTE	
	ACCEPT "values used are estimations or approximated" WTTE for A1 **DO NOT ACCEPT** "inaccurate because I rounded" WTTE accurate regardless their justification just rounding their results	**DO NOT ACCEPT** "inaccurate because I rounded" WTTE accurate regardless their justification just rounding their results

Q7 Task 3

Glossary for task 3

Term used	Clarification
General rule	Rule in terms of only n (if they use x, it is still general rule but penalise in notation)
The general rule	The correct general rule in terms of only n (if they use x, it is still the general rule but penalise in notation)
Their general rule	valid attempt for the general rule but in terms of only n (if they use x, it is still their general rule but penalise in notation)
Their rule	Correct rule not in terms of only n
Recursive rule	$A = \dfrac{A_{n-1}}{4} OE$

We accept subsequent use of their general rule provided it is of similar complexity. In general, the complexity of the rule depends on its form. The table below shows examples.

The general rule	ACCEPT for their general rule	DO NOT ACCEPT for their general rule
$T = 8n - 8$	Linear	
$A = \dfrac{8}{2^n}$	Quadratic or exponential	Linear

7		Answers	Notes		Total
	a	Correctly place 32 and 40	5	32	**1**
			6	40	

b	.1 correctly describe one pattern for *T* in words .2 correctly describe a second pattern for *T* in words	ACCEPT complete terminology only, for example (below are four different descriptions) divisible by 8, multiples of 8 Increasing by 8, adds 8, goes up by 8, moving up by 8 difference 8, common difference 8, linear with difference 8, arithmetic with difference 8 Second difference zero DO NOT ACCEPT incomplete terminology, for example: Arithmetic, linear, increasing by a constant, constant difference DO NOT ACCEPT general description, for example: Integers, whole numbers, positive, even numbers, divisible by 2, multiples of 2, multiples of 4 DO NOT ACCEPT the rule in words or description related to n for example: 8 times n minus 8, n multiplied by 8 minus 8, WTTE **Note, in the case when they have more than two different patterns:** If two are accepted and the rest are all correct: award 2 marks Ex: even numbers and adds 8 and Second difference is zero and it is 8 times n then subtract 8. Ex: multiples of 2 and 4 and linear and increases by 8 and divisible by 8 If two are accepted and any of the rest is incorrect: award 1 mark Ex: <u>increases by 2</u> and Second difference is zero and it is divisible by 8 If only one is accepted, ignore the rest and award 1 mark	**2**
c	.1 the correct general rule .2 the correct simplified general rule with correct notation for *T* in terms of *n*.	.1 $8n - 8$ or $8(n-1)$ or $T = 8 \times n - 8$ or $(T =)8 * n - 8$ or $T = 8 \times (n-1)$ or $T = 0 + 8(n-1)$ .2 $T = 8n - 8$ or $T = 8(n-1)$ .2 ACCEPT using Tn instead of T .2 ACCEPT $T = n8 - 8$ or $T = (n-1)8$ .2 ACCEPT using t and N DO NOT ACCEPT description in words	**2**

	d	.1 correctly substitute $n \geq 5$ into their general rule (from 7c or 7b)	.1 Ex: $8 \times 5 - 8$	**3**
		.2 correctly calculate their value of T after substituting $n \geq 5$	.2 Ex: 32 (for $n = 5$)	
		.3 recognise that their correctly calculated value of T is the same as their predicted value	.3 "the same as when we continue the pattern" WTTE and states how Ex: for $n=7$, 48 is obtained from pattern of adding 8 to 40 .3 ACCEPT if their value from .2 is the same as their value in the table in part a) or seen here in part d) Ex: we see their calculated $T=32$ and we see $T=32$ in their table	
	e	Correctly write $\dfrac{1}{4}$ and $\dfrac{1}{8}$ as powers of 2 2^{-5} AG	$2^{-2}(\times)2^{-3}$ or 2^{-2-3} ACCEPT seeing $\dfrac{1}{2^2} \times \dfrac{1}{2^3}$ or $\dfrac{1}{2 \times 2} \times \dfrac{1}{2 \times 2 \times 2}$ or $\dfrac{1}{32}$ AND $\dfrac{1}{2^5}$ or $\dfrac{1}{2 \times 2 \times 2 \times 2 \times 2}$ ACCEPT working with denominator. Ex: seeing $4 \times 8 = 32$ AND $32 = 2^5$ ACCEPT notation errors. Ex: $\dfrac{1}{4}$ times $\dfrac{1}{8} = \dfrac{1}{32}$ AND 2^5 is 32 ACCEPT equalities on one line. Ex: $\dfrac{1}{4} \times \dfrac{1}{8} = \dfrac{1}{32} = \dfrac{1}{2 \times 2 \times 2 \times 2 \times 2}$ DO NOT ACCEPT the justification $\dfrac{1}{4} \times \dfrac{1}{8} = \dfrac{1}{32}$ AND $2^{-5} = \dfrac{1}{32}$ DO NOT ACCEPT $\dfrac{1}{4} \times \dfrac{1}{8} = -32$ or 32 DO NOT ACCEPT working with decimals. Ex: $\dfrac{1}{4} \times \dfrac{1}{8} = \dfrac{1}{32} = 0.03125$ AND $2^{-5} = 0.03125$	**1**

Q7f 23 marks

5	$\frac{1}{8}$	$\frac{1}{16}$		$\frac{1}{128}$	2^{-7}
6	$\frac{1}{16}$	$\frac{1}{32}$		$\frac{1}{512}$	2^{-9}
7	$\frac{1}{32}$	$\frac{1}{64}$		$\frac{1}{2048}$	2^{-11}
8	$\frac{1}{64}$	$\frac{1}{128}$		$\frac{1}{8192}$	2^{-13}

Mark	1	2	3
Predictions (P)	Correctly predict **two** terms for L or W	Correctly predict **two** terms for L or W <u>and</u> correctly predict **one** term for A	Correctly predict **two** terms for L <u>and</u> **two** terms for W **AND**
	OR Correctly predict **one** term for A	**OR** Correctly predict **two** terms for A	Correctly predict **two** terms for A
	Ignore additional incorrect predictions **ACCEPT** fractions or index form **ACCEPT** in the table or in the response box **DO NOT ACCEPT** decimals	Ignore additional incorrect predictions **ACCEPT** fractions or index form **ACCEPT** in the table or in the response box **DO NOT ACCEPT** decimals	Ignore additional incorrect predictions **ACCEPT** fractions or index form **ACCEPT** in the table or in the response box **DO NOT ACCEPT** decimals

Examples of rules in equivalent forms:

$$L = \frac{4}{2^n} = 2 \times (\frac{1}{2})^{n-1} = \frac{1}{2^{n-2}} = 2^{2-n}$$

$$W = \frac{2}{2^n} = 1 \times (\frac{1}{2})^{n-1} = \frac{1}{2^{n-1}} = 2^{1-n}$$

$$A = \frac{8}{4^n} = 8 \times (\frac{1}{4})^n = 2 \times (\frac{1}{4})^{n-1} = 2 \times 4^{1-n} = \frac{2^3}{2^{2n}} = \frac{1}{2^{2n-3}} = 2^{3-2n}$$

Mark	1	2	3	4	5
Description (D) ACCEPT notation errors, non-simplified rule, or rule in words but penalize in notation (N)	Correctly describe a pattern in words for A Ex: Multiply by $\frac{1}{4}$ OE, Divide by 4, Divide by 2 two times ratio $\frac{1}{4}$ OE , Denominators multiply by 4 The power decreases by 2 or common difference 2 **OR** Correct recursive rule for A $A = \frac{A_{n-1}}{4}$ OE or the rule $A = 2^n$ **OR** Valid attempt to write down a general rule for L or W Ex: 2^n or 4^n seen in rule for L or W ACCEPT if they correctly describe a pattern in words or recursive rule for L or W Ex: Multiply by $\frac{1}{2}$ OE , Divide by 2 Denominators are multiples of 2 or divisible by 2 or even numbers or 2 to the power something $L = \frac{L_{n-1}}{2}$ or $W = \frac{W_{n-1}}{2}$ OE **Ignore additional incorrect patterns** **DO NOT ACCEPT** Exponential, geometric, arithmetic sequence, denominators are multiples of 2 or even numbers , $A=L$xW , it is 2 to the power something, A is decreasing	Correctly describe a pattern in words for A AND Correct recursive rule for A or rule $A = 2^n$ or Valid attempt to write down a general rule for L or W **OR** Valid attempt to write down a general rule for A Ex: only denominator correct in their general rule for A Ex: their general rule for A is in a form 2^{an+b} Ex: $A = 4^n$ but do not accept $A = 2^n$	Correctly describe a pattern in words or recursive rule for A AND Valid attempt to write down a general rule for A **OR** Correctly write down general rule for L or W $L = \frac{4}{2^n} OE$ $W = \frac{2}{2^n} OE$ (see examples above)	Correctly describe a pattern in words or recursive rule for A AND Correctly write down general rule for L or W **OR** Correctly write down the general rule for A $A = 2^{3-2n}$ OE (see examples above)	Correctly describe a pattern in words or recursive rule for A **AND** Correctly write down the general rule for A

Mark	1	2	3
Testing (T) ACCEPT transforming into decimals when testing	Attempt to test their general rule for A using $n \le 4$ Ex: Substitute in their general rule for A value of $n \le 4$ **OR** Correctly test their described pattern or their rule (e.g. recursive rule) or their rule for Lor$W$ in terms of n **DO NOT ACCEPT** testing $A=LxW$	Correctly test their general rule for A using $n \le 4$ Ex: Correctly calculate their value for A in their general rule using $n \le 4$ **AND** Recognise that their correctly calculated value for A is the same as the given value. **ACCEPT** seeing their correctly calculated value for A, as fraction or decimal, and the given value in the table being equal **ACCEPT** testing only $an+b$ when they have 2^{an+b} in their general rule for A	
Verifying (V) ACCEPT transforming into decimals when verifying	Attempt to verify their general rule for A using $n \ge 5$ Ex: Correctly substitute in their general rule for A value of $n \ge 5$ **OR** Correctly verify their described pattern or their rule (e.g. recursive rule) or their rule for Lor$W$ in terms of n Correctly verify their described pattern or their rule **DO NOT ACCEPT** verifying $A=LxW$	Correctly calculate their value for A in their general rule using $n \ge 5$ **ACCEPT** verifying only $an+b$ when they have 2^{an+b} in their general rule for A	Correctly calculate their value for A in <u>the general rule</u> using $n \ge 5$ **AND** Recognise that their correctly calculated value for A is the same as the correct predicted value obtained by continuing the pattern **ACCEPT** seeing their correctly calculated value for A, as fraction or decimal, and the correctly predicted value in the table being equal **ACCEPT** verifying only $an+b$ when they have 2^{an+b} in the general rule for A

Mark	1	2	3	4
Justify/ proof (J)	**ACCEPT only if D1 is achieved**	**ACCEPT only if D4 is achieved**	**ACCEPT only if D4 is achieved**	**ACCEPT only if D4 is achieved**
	Attempt to use the geometric or arithmetic sequence to justify <u>their</u> general rule aligned with their notation for <u>their</u> general rule or recursive rule for A or the general rule for L or W or attempt to substitute and find parameters (see examples in next page)	Correctly use geometric or arithmetic sequence to justify <u>the</u> general rule aligned with their notation for <u>the</u> general rule for A, or correctly substitute and find parameters (see examples in next page)	Attempt to justify geometrically the general rule for A Use the product of correct L and W rules <u>without complete simplification</u> Ex: $(A = L \times W =) \dfrac{4}{2^n} \times \dfrac{2}{2^n} = \dfrac{8}{2^{2n}}$ $(A = L \times W =) \dfrac{2}{2^{n-1}} \times \dfrac{1}{2^{n-1}} = \dfrac{2}{4^{n-1}}$	Correctly justify geometrically the general rule for A Use the product of correct L and W rules <u>with complete simplification</u> : They show how the product of the correct L and W rules simplifies to 2^{3-2n} or $\dfrac{1}{2^{2n-3}}$ Ex: $(A = L \times W =)$ $2^{2-n} \times 2^{1-n} = 2^{3-2n}$ or $(A = L \times W =)$ $\dfrac{2}{2^{n-1}} \times \dfrac{1}{2^{n-1}} = \dfrac{2}{2^{2n-2}} = \dfrac{1}{2^{2n-3}}$
	OR Weak attempt to justify <u>their</u> general rule for A geometrically by using the product of <u>incorrect L</u> and <u>W rules</u> ACCEPT: L is halved and W is halved so A is divide 4 WTTE and we see 4 in the denominator of their general rule	**OR** Weak attempt to justify <u>the</u> general rule for A geometrically by using the product of <u>incorrect L</u> and <u>W rules</u> or by using correct L and W rules but with mistakes in simplification ACCEPT A=LxW **seen as justification** only if the correct rules for L and W are mentioned		
	OR Substitute at least two other values of n in their general rule for A or L or W and say they are the same or hence the rule works WTTE			

	EXAMPLES ALLOWING J1 (ACCEPT ONLY IF D1 IS ACHIEVED) Attempt to use the geometric or arithmetic sequence to justify their general rule aligned with their notation for their general rule or recursive rule for A or the general rule for L or W, OR attempt to substitute and find parameters	EXAMPLES ALLOWING J2 (ACCEPT ONLY IF D4 IS ACHIEVED) Correctly use geometric or arithmetic sequence to justify the general rule aligned with their notation for the general rule for A, OR correctly substitute and find parameters
Using geometric sequence	Ex that must to be seen as justification other than describing pattern: Every time we multiply by $\frac{1}{4}$ so it makes sense to have 4 in my general rule WTTE and we see 4 or $\frac{1}{4}$ in their general rule or recursive rule for A	Ex that must to be seen as justification other than describing pattern: First term is 2 then we multiply once by $\frac{1}{4}$ to get 2nd term and we multiply twice by $\frac{1}{4}$ to get 3rd term and we multiply $(n-1)$ times by $\frac{1}{4}$ to get nth term WTTE and we see $(\frac{1}{4})^{n-1}$ in the general rule for A
	Ex: Ratio or $r = 2$ and we see 2 to the power something including n in the denominator of their general rule for A Ex: Ratio or $r = 4$ and we see 4 to the power something including n in the denominator of their general rule for A ACCEPT $(A =) r^{n-1}$ and their general rule for A where r is the correct value For L or W general rule, ACCEPT $(L \text{ or } W =) ar^{n-1}$ and the general rule for L or W where a and r are the correct values	Ex: First term or $a = 2$ and ratio or $r = \frac{1}{4}$ and the general rule $2 \times (\frac{1}{4})^{n-1}$ Ex: First term or $a = \frac{1}{2}$ and ratio or $r = 4$ and the general rule $\frac{1}{\frac{1}{2} \times 4^{n-1}}$ Ex: First term or $a = 1$ and ratio or $r = 4$ and the general rule $\frac{2}{4^{n-1}}$ ACCEPT $(A =) r^{n-1}$ and the general rule for A where a and r are the correct values
Using arithmetic sequence	State meaning of a in 2^{an} Ex: The power of 2 decreases by 2 or difference -2 and they have 2^{-2n} in their general rule for A WTTE Ex: The power of 2 increases by 2 or difference 2 and they have $\frac{1}{2^{2n}}$ in their general rule for A	State meaning of a and b in 2^{an+b} Ex: power of 2 decreases by 2 and first term power is 3 more than -2 or always 3 more WTTE Ex: First term or $a = 1$ and difference $d = -2$ and the general rule 2^{3-2n} Ex: First term or $a = -1$ and difference $d = 2$ and the general rule $\frac{1}{2^{2n-3}}$
Using parameters substitution	Assume $b \times c^n$ and substitute $c = \frac{1}{4}$ and a value for A and n then find incorrect b	Assume $b \times c^n$ and substitute $c = \frac{1}{4}$ and a value for A and n then find $b=8$
	Assume 2^{an+b} and substitute two points and find $a = -2$ or $b = 3$	Assume 2^{an+b} and substitute two points and find $a = -2$ and $b = 3$

Mark	1	2	3
Notation and terminology (N)	**ACCEPT only if D1 achieved** Correct notation of <u>their</u> general rule for *A* or the rule for *L* or *W* Ex: (ACCEPT non-simplified rules) $A = (\frac{1}{4})^n$ or $A = 0.25^n$ $L = \frac{4}{2^n}$ $W = \frac{2}{2^n}$ **OR** The notation of <u>the general</u> rule includes errors or not simplified or in words Ex: $A = 2\text{^}(3\text{-}2n)$, $A = \frac{1}{2^{2 \times n - 3}}$, $A = \frac{1}{2^{2 \times n - 3}}$ or $A = \frac{2^3}{2^{2n}} OE$, $A = \frac{8}{4^n} OE$, $A = 2 \times 0.25^{n-1}$ or 8 over 4 power *n* , The rule for *A* is 2^{3-2n} **OR** Correctly describe a pattern in words or recursive rule for *A* Ex: $A = \frac{A_{n-1}}{4}$ **DO NOT ACCEPT** if they don't have any rules and they don't describe any patterns	**ACCEPT only if D4 achieved** Correct notation of <u>the general</u> rule for *A* in simplest form $A = 2^{3-2n}$ or $A = 2^{(3-2n)}$ ACCEPT $A = \frac{1}{2^{(2n-3)}}$ or $A = 2^{-2n+3}$ **OR** The notation of <u>the general</u> rule includes errors or not simplified or in words (see examples in N1) <u>and</u> Correctly describe a pattern in words for *A* **ACCEPT** using U_n instead of *A* only if they mention that $A = U_n$ **DO NOT ACCEPT** using * for multiplication using / for division using ^ for power Using x instead of *n* **DO NOT ACCEPT** if they don't have the general rule for *A*	**ACCEPT only if D5 achieved** Correct notation of <u>the general</u> rule for *A* in simplest form (see examples in N2) **AND** Correctly describe a pattern in words for *A* **ACCEPT** using U_n instead of *A* only if they mention that $A = U_n$ **DO NOT ACCEPT** using * for multiplication using / for division using ^ for power Using x instead of *n* **DO NOT ACCEPT** if they don't have the general rule for *A*

Mark	1	2	3

Communication (L)
 	
Organisation and coherence
Can be awarded even if there are errors
 	
For items:
Describing pattern and writing rule can be considered an item even if D0 awarded
	At least three from the following **items** are seen:
	• describe a pattern in words
	• write a rule
	• test their general rule or rule or recursive rule or pattern (at least T1)
	• verify their general rule or rule or recursive rule or pattern (at least V1)
	• justify their general rule or rule or recursive rule or pattern (at least J1)
		ACCEPT only if they have a general rule
		At least four of the following **items** are seen:
		• describe a pattern in words
		• write a general rule
		• test their general rule (at least T1)
		• verify their general rule (at least V1)
		• justify their general rule (at least J1)
		
		AND
		For coherence, they identify the processes correctly. **At least one** from the following:
		• test
		• verify
		• justify
		
		Ex:
		   • **For test:**
		they say "test" and they substitute in their general rule value(s) of $n \leq 4$ only
		
		   • **For verify:**
		they say "verify" and they substitute in their general rule value(s) of $n \geq 5$ only
		
		   • **For justify:**
		They say "justify" and they write a justification
		
		Note for coherence:
		If they say "test and verify" and they substitute in their general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process
			ACCEPT only if they have the general rule for A
			The following two items must be seen :
			• write <u>the general rule for A</u>
			• justify <u>the general rule</u> (at least J2)
			AND
			At least two of the following **items** are seen:
			• describe a pattern or rule in words
			• test <u>the general rule</u> (at least T1)
			• verify <u>the general rule</u> (at least V1)
			
			AND
			For coherence, they identify the processes correctly. **At least two** from the following:
			• test
			• verify
			• justify
			
			Ex:
			   • **For test:**
			they say "test" and they substitute in the general rule for A value(s) of $n \leq 4$ only
			
			   • **For verify:**
			they say "verify" and they substitute in the general rule for A value(s) of $n \geq 5$ only
			
			   • **For justify:**
			They say "justify" and they write a justification
			
			Note for coherence:
			If they say "test and verify" and they substitute in their general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process